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SHORT COMMUNICATION

# APPLICATION OF TRAVELING SALESMAN PROBLEM (TSP) FOR DECISION OF OPTIMAL PRODUCTION SEQUENCE

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**Abstract** – In the present study a reliable and structural decision system for production sequence of polymeric products is developed. Minimization of the amount of off-specs is the main objective in the decision of production sequence to maximize profit. Off-specs are generated when the production sequence of polymeric products is changed. The amount of off-specs depends on changes of product grades. In the present study we applied the traveling salesman problem (TSP) to achieve optimal decision of production sequence. To solve the optimal decision problem formulated by TSP, we employed three different approaches such as Branch and Bound (B&B) method, Dynamic Programming (DP) method and Hopfield Neural Network (HNN) method. Production sequences computed based on the actual plant off-spec data were compared with the sequences employed in the actual plant operation. From the comparison the decision method proposed in the present study showed increased profits and reduced off-specs.

*Key words: Production Sequence, Off-spec, TSP, B&B, DP, HNN*

## INTRODUCTION

In the production of various petrochemical products such as polymers the decision of optimal production sequence is by far the most important to increase profit. The decision problem of production sequence especially in batch plants can be solved based on various criteria such as minimization of the mean flow time, shortest processing time, mean of weighted flow time, and minimization of earliness and tardiness [Nicolas, 1986; Baker and Ashley, 1989].

In the actual operation of plants producing polymeric products with various grades, considerable amounts of off-specification (off-spec) are generated with changes of grades. The off-spec causes delay in the process operation, loss of valuable feed materials and decrease in the profit. Therefore the minimization of the amount of off-spec can be an important criterion in the decision of production sequence. The decision problem of production sequence based on the minimization of the amount of off-spec can be effectively solved by Traveling Salesman Problem (TSP).

So far many computational techniques have been reported to handle TSP. Among them Branch and Bound (B&B) [Lawer and Wood, 1966] and Dynamic Programming (DP) [Bellman, 1957; Egbelu, 1990] showed practical applications. Recently several researchers proposed use of the Hopfield Neural Network (HNN) [Hopfield and Tank, 1985] [Samad and Haper, 1990].

In the present study the decision problem of production sequence in a petrochemical plant producing various polyethylene products with different grades was investigated. We focused on the minimization of the total amount of off-spec occurred in the actual operation. We assumed that products with the same grade be produced once a month and did not consider inventory cost nor interest rate. Also it was assumed that the product value of off-spec is 10% less than that of pure product. The TSP was employed and the three computational techniques described before were used to solve the decision problem. The sequences obtained by the present methods were compared with those used in the plant. We could verify that the amount of off-spec was considerably decreased and the profit was increased by the present production sequences.

## FORMULATION OF TSP

The traveling salesman problem (TSP) is based on the minimization of the distances between various locations where the salesman travels. Namely, TSP is a problem that a salesman, starting in one city, visits each of n−1 other cites once and only and return to start, so seeks tour to minimize the total distance or cost. This TSP is similar to production sequence decision problem which is to minimize total off-spec generations caused by changes of product grade. The solution of TSP is very complicated due to the existence of many possible cases. The TSP can be formulated as

$$\text{Minimize: } Z=\sum_{i}^{n}\sum_{j}^{n} x_{ij}\, d_{ij} \tag{1}$$

$$\text{Subject to: } \sum_{i}^{n} x_{ij}=1,\quad \sum_{j}^{n} x_{ij}=1 \tag{2}$$

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$$\text{where } x_{ij} = \begin{cases} 1\ ; & \textit{salesman travels from city } i \textit{ to city } j \\ 0\ ; & \textit{otherwise} \end{cases} \tag{3}$$

A tour, x, forms a circuit connecting each city once and only once and can be represented as a set of n ordered city pairs as

$$x = \{(i_1, i_2), (i_2, i_3), (i_3, i_4), \cdots, (i_{N-1}, i_N), (i_N, i_1)\} \tag{4}$$

Each (i, j) represents a node of the tour. The objective is to find the order of tour which minimizes total distance. In the present study the above TSP was used with slight modification including replacement of d by off-specs.

## SOLUTION OF TSP

To solve the TSP, we used three different methods: Branch and Bound (B&B) method, Dynamic Programming (DP) method and Hopfield Neural Network (HNN) method. These methods are especially useful in the solution of the TSP. The choice of a suitable method may depend on required products and constraints.

### 1. Branch and Bound(B&B) Method

B&B method is characterized by lower bound and branching. In this method the set of all tours is broken into smaller subsets and lower bound is used to decide the best tour based on the computed distance.

In Fig. 1, the node $(\overline{i, j})$ represents all tours that include the city pair (i, j) and the node (i, j) represents all the other tours. The node $(\overline{k, l})$ represents all tours that include (i, j) but not (k, l), whereas (k, l) represents all tours that include both (i, j) and (k, l).

The solution procedure of B&B method can be summarized as following:

(1) Construct a matrix $D=[d_{ij}]$.
(2) Perform a row-reduction on D.
(3) Perform a column-reduction on D.

Reduction procedure is subtracting the smallest element in the same row (or column) from all row (or column) elements. At this stage, matrix D becomes reduced matrix D, which has at least one zero.

(4) Calculate the sum S of the reducing constants. S constitutes a lower bound.

The total distances to travel, Z, is given by

$$Z = Z_r + S \tag{5}$$

(5) Calculate $\theta_{ij}$ for each zero element of $D_r$.

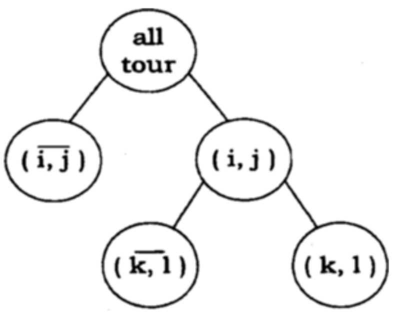


**Fig. 1. Tree of tour: search through branching.**

(6) Consider branching from node (i, j) which has maximum value of $\theta_{ij}$.

The lower bound of $(\overline{i, j})$, LB $(\overline{i, j})$, is given by

$$LB\ (\overline{i, j}) = \text{LB (all tours)} + \theta_{ij} \tag{6}$$

(7) Eliminate row i, column j and set $d_{ji}=\infty$ to avoid formation of subsequence.

(8) List eliminated row and column then arrange listed pairs like (4).

(9) If pairs do not arranged, go back to step 2. Otherwise, whenever x is arranged like (4), set $d_{i_3 i_1}=\infty$, $d_{i_4 i_1}=\infty$, $\cdots$, etc. to avoid formation of subsequence and go back to step 2.

### 2. Dynamic Programming (DP) Method

In the present study DP is used with the assumption that the production sequence starts with a dummy operation and ends with a dummy operation. In DP method an action is selected at each decision point to minimize total current cost (i. e., distance) as well as the best future cost. The DP method is useful especially when constraints are include in the problem. The DP algorithm can be summarized as following:

(1) Compute the amount of off-spec according to (7) using the distance table.

$$F_1(j\sigma_0) = C_{ji_{n-1}} \tag{7}$$

(2) Determine the partial sequence according to (8)

$$F_J(j\sigma_{j-1}) = \min\{\overline{F}(ji, i\sigma_{J-2}), \overline{F}(ij, j\sigma_{J-2})\} \tag{8}$$

$$\text{where } j = 1, 2, \cdots, n,\ j \notin \sigma_{j-1},\ i \in \sigma_{J-1}$$

$$\overline{F}(ji, i\sigma_{J-2}) = C_{ji} + \sum_{i,k \in \sigma_{i-1}} C_{ik}$$

(3) Perform recursive calculation of $F_J$ $(3 \le J \le n)$ by using the recursive relation:

$$F_J(j\sigma_{j-1}) = \min\{\overline{F}(jik, k\sigma_{J-2}), \overline{F}(ikj, i\sigma_{J-2}), \overline{F}(kji, i\sigma_{J-2}), \overline{F}(ijk, k\sigma_{J-2}), \overline{F}(kij, k\sigma_{J-2}), \overline{F}(jki, i\sigma_{J-2})\} \tag{9}$$

$$\text{where } j=1, 2, \cdots, n,\ j \notin \sigma_{j-1},\ i \in \sigma_{J-1}$$

At this stage, the algorithm progressively moves stage by stage using backward recursion until the final stage is reached.

(4) The optimal sequence is found when J=n+1 as

$$F_{n+1}(i_0\,\sigma_n) = \min\{\overline{F}(i_0\,i, i\,\sigma_{n-1})\},\ i \in \sigma_n \tag{10}$$

### 3. Hopfield Neural Network (HNN) Method

Many combinatorial optimization problems can be mapped onto neural networks by constructing suitable energy function and then transforming the minimization problem into associated systems of differential or difference equations. The energy function proposed by Hopfield and Tank [Hopfield and Tank, 1985] to solve TSP is defined by

$$\begin{aligned} E = & \frac{A}{2}\sum_x \sum_i \sum_{j \neq i} v_{x,i}\, v_{x,j} \\ & + \frac{B}{2}\sum_i \sum_x \sum_{y \neq x} v_{x,i}\, v_{y,i} \\ & + \frac{C}{2}\,[N - \sum_x \sum_i v_{x,i}]^2 \\ & + \frac{D}{2}\,[\sum_x \sum_{y \neq x} \sum_i d_{x,y}\, v_{x,i}\,(v_{y,i+1} + v_{y,i-1})] \end{aligned} \tag{11}$$

The positive coefficients A, B, C and D are penalty constants. They are multiplied by corresponding constraints terms to enforce the neural network to converge to a state which denotes a valid tour. Hopfield neural network is constructed by connecting a large number of simple processing element (artificial neurons) to each other. Each unit receives an external input signal of (12).

$$I_{xi} = C\ M \tag{12}$$

where the parameter M is usually taken to be somewhat larger than n. The output signal is given by applying the well-known sigmoid function.

$$v_i = g(u_i) = 0.5[1+\tanh(\alpha u_i)] \tag{13}$$

Procedure of HNN can be summarized as following:

(1) Construct distance matrix.
(2) Initialize activation of all unit and $\triangle t$.
(3) Calculatc weight matrix w which is given by

$$w(x, i{:}y, j) = -A\,\delta_{xy}\,(1-\delta_{xy}\,(1-\delta_{ij}) - B\delta_{ij}\,(1-\delta_{xy})$$
$$-C - D\delta_{xy}\,(\delta_{i,j+1} + \delta_{i,j-1}) \tag{14}$$

(4) Initialize input vector.
(5) Change activity to a selected unit.

$$u_{x,i}(\text{new}) = u_{x,i}(\text{old}) + \triangle t\,[-u_{x,i}(\text{old}) - A\sum_{j\neq i} v_{x,j} - B\sum_{y\neq x} v_{y,i}$$
$$-C\,(N-\sum_x\sum_j v_{x,j}) - D\sum_{y\neq x,} d_{x,y}\,(v_{y,i+1} + v_{y,i-1})] \tag{15}$$

(6) Apply output function according to (13).
(7) Calculate activation matrix. Activities to obtain $u_{x,i}$ (new) is updated synchronously $n^2$ times through step 5 and 7.
(8) Check a given criterion. If the criterion is satisfied, go back to step 4.

## DECISION OF PRODUCTION SEQUENCE OF POLYETHYLENE PRODUCTS

The methods described so far were applied to decide production sequence of polyethylene products. As stated before, minimization of the amount of off-specs is the prior purpose in the decision of production sequence. Table 1 shows the off-specs quantities generated in the actual plant operation. In the table, P-01, P-02, ⋯, P-16b denote grades of various polyethylene products and the values present average generation of off-spec quantities according to grade changes from one product to another production. Besides on the minimization of off-specs, the grades which have lower inventory level than safe stock should be produced first.

The production sequence should maximize profit while satisfying imposed constraints. The profit function is given by

$$P = \sum_i^N[\{PV_i - PC_i\} \times PQ_i] - \sum_i^N[\{PV_i \times 0.9 - PC_i\} \times TCPQ] \tag{16}$$

The required informations to solve the problem include inventory level, shortage cost, interest rate, etc. But in the present study, we did not incorporate shortage cost and interest rate for simplicity. And we concentrated on total generation of off-specs for optimization so that other elements were neglected. Actually the shortage cost and interest rate were found to have little effect on the total profit.

## RESULTS AND DISCUSSIONS

The computational results of the present study were compared with the production sequences employed in the actual plant. The plant has two different production trains and the off-spec quantities shows large variances depending on trains and times (month). Thus we adapted data during three different months (August, 1996, September, 1996 and October, 1996) for each train in the computation. The production sequences determined by the present study are shown in Table 2-5 as well as the sequences used in the actual operations.

As can be seen, all the three methods studied in the present study show better results than the sequences employed in the

**Table 1. Average generation of off-specs according to grade changes (MT)**

| | | | To | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Grade | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| | 1 | P-01 | – | 0 | 25 | 30 | 35 | 45 | 75 | 110 | 70 | 80 | 60 | 60 | 55 | 55 | 100 | 100 |
| | 2 | P-02 | 0 | – | 25 | 30 | 35 | 45 | 75 | 110 | 70 | 80 | 60 | 60 | 55 | 55 | 100 | 100 |
| | 3 | P-03 | 20 | 20 | – | 20 | 35 | 40 | 65 | 100 | 70 | 60 | 40 | 60 | 60 | 60 | 90 | 90 |
| | 4 | P-04 | 30 | 30 | 20 | - | 35 | 40 | 65 | 100 | 70 | 60 | 40 | 60 | 60 | 60 | 90 | 90 |
| | 5 | P-05 | 60 | 60 | 35 | 35 | - | 30 | 70 | 100 | 100 | 60 | 60 | 45 | 40 | 40 | 60 | 60 |
| | 6 | P-06 | 60 | 60 | 40 | 45 | 35 | – | 65 | 100 | 120 | 90 | 50 | 50 | 40 | 40 | 90 | 90 |
| | 7 | P-07 | 70 | 70 | 60 | 60 | 45 | 45 | – | 80 | 140 | 110 | 60 | 60 | 60 | 60 | 100 | 100 |
| From | 8 | P-08 | 70 | 70 | 60 | 60 | 55 | 40 | 80 | – | 140 | 110 | 60 | 50 | 45 | 45 | 100 | 100 |
| | 9 | P-09 | 50 | 50 | 75 | 75 | 80 | 55 | 140 | 110 | – | 80 | 55 | 55 | 50 | 50 | 100 | 100 |
| | 10 | P-10 | 70 | 70 | 40 | 40 | 80 | 55 | 140 | 110 | 70 | – | 45 | 50 | 40 | 40 | 80 | 80 |
| | 11 | P-11 | 60 | 60 | 60 | 60 | 80 | 45 | 85 | 110 | 60 | 60 | – | 50 | 50 | 50 | 90 | 90 |
| | 12 | P-12 | 90 | 90 | 55 | 55 | 45 | 55 | 70 | 130 | 90 | 55 | 70 | – | 0 | 0 | 40 | 40 |
| | 13 | P-13 | 90 | 90 | 45 | 45 | 40 | 50 | 70 | 130 | 90 | 45 | 70 | 0 | – | 0 | 40 | 40 |
| | 14 | P-14 | 90 | 90 | 45 | 45 | 40 | 50 | 70 | 130 | 90 | 45 | 70 | 0 | 0 | – | 40 | 40 |
| | 15 | P-15a<br>P-15b | 130 | 130 | 85 | 85 | 70 | 80 | 180 | 150 | 130 | 85 | 90 | 35 | 35 | 35 | – | 0 |
| | 16 | P-16a<br>P-16b | 130 | 130 | 85 | 85 | 70 | 80 | 180 | 150 | 130 | 85 | 90 | 35 | 35 | 35 | 0 | – |

**Table 2. Production sequences (1996. 8., train # 1)**

| | Plant | | Present methods by | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | B&B | | DP | | HNN | |
| Production sequence | P-03 | (20) | P-03 | (20) | P-03 | (20) | P-03 | (20) |
| | P-01 | (35) | P-01 | (35) | P-01 | (35) | P-01 | (35) |
| | P-05 | (70) | P-05 | (40) | P-05 | (40) | P-05 | (40) |
| | P-07 | (60) | P-13 | (0) | P-13 | (0) | P-14 | (0) |
| | P-14 | (0) | P-14 | (0) | P-12 | (0) | P-13 | (0) |
| | P-12 | (0) | P-12 | (70) | P-14 | (70) | P-12 | (70) |
| | P-13 | (50) | P-07 | (45) | P-07 | (45) | P-07 | (45) |
| | P-06 | (45) | P-06 | (45) | P-06 | (45) | P-06 | (45) |
| | P-04 | (20) | P-04 | (20) | P-04 | (20) | P-04 | (20) |
| | P-03 | | P-03 | | P-03 | | P-03 | |
| Off-specs (MT) | 300 | | 275 | | 275 | | 275 | |
| Change profit (won) | – | | +21,926,000 | | +13,037,000 | | +10,063,000 | |

( ): change generation of off-specs

actual operation. The amounts of off-specs were decreased and the profits were increased by the present methods. This fact demonstrates the effectiveness of the TSP in the decision of optimal production sequence. But there remains the problem of selection method shows its own characteristics. B&B method has fewer iterations relatively, because iterations are dependent on the number of selected product. Comparing many iterations of HNN method determined dependency on user's option, as if iteration is user's option, it is more than the number of selected product, which means B&B method has advantage in terms of computing time. But computing time of these method is shorter than a few seconds due to computer's development and production of some dozen of product. DP method must possess the sequences calculated until previous stage and in the HNN method, mediatory space like weight matrix and activation matrix needs during DP performance. So DP and HNN methods require larger memories compared with B&B method. But DP can handle constraints

**Table 3. Production sequences (1996. 8., train #2)**

| | Plant | | Present methods by | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | B&B | | DP | | HNN | |
| Production sequence | P-14 | (0) | P-14 | (45) | P-14 | (45) | P-14 | (40) |
| | P-13 | (90) | P-10 | (70) | P-10 | (70) | P-15a | (35) |
| | P-09 | (80) | P-09 | (100) | P-09 | (50) | P-13 | (45) |
| | P-10 | (80) | P-15a | (35) | P-13 | (40) | P-10 | (70) |
| | P-15a | (35) | P-13 | (0) | P-15a | (35) | P-09 | (50) |
| | P-14 | | P-14 | | P-14 | | P-14 | |
| Off-specs (MT) | 285 | | 250 | | 240 | | 240 | |
| Change profit (won) | – | | +20,273,000 | | +15,817,000 | | +18,144,000 | |

( ): change generation of off-specs

**Table 4. Production sequences (1996. 9., train #1)**

| | Plant | | Present methods by | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | B&B | | DP | | HNN | |
| Production sequence | P-02 | (0) | P-02 | (35) | P-02 | (0) | P-02 | (0) |
| | P-01 | (35) | P-05 | (30) | P-01 | (30) | P-01 | (35) |
| | P-05 | (40) | P-06 | (40) | P-04 | (35) | P-05 | (30) |
| | P-14 | (0) | P-13 | (0) | P-05 | (30) | P-06 | (40) |
| | P-12 | (0) | P-14 | (0) | P-06 | (40) | P-14 | (0) |
| | P-13 | (50) | P-12 | (55) | P-13 | (0) | P-13 | (0) |
| | P-06 | (45) | P-04 | (20) | P-12 | (0) | P-12 | (55) |
| | P-04 | (20) | P-03 | (20) | P-14 | (45) | P-04 | (20) |
| | P-03 | (20) | P-01 | (0) | P-03 | (20) | P-03 | (20) |
| | P-01 | | P-02 | | P-02 | | P-02 | |
| Off-specs (MT) | 210 | | 200 | | 200 | | 200 | |
| Change profit (won) | – | | +4,408,000 | | +4,941,000 | | +3,957,000 | |

( ): change generation of off-specs

**Table 5. Production sequences (1996. 10., train #1)**

| | Plant | | Present methods by | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | B&B | | DP | | HNN | |
| Production sequence | P-01 | | P-01 | | P-01 | | P-01 | |
| | | (35) | | (0) | | (30) | | (30) |
| | P-05 | | P-02 | | P-04 | | P-04 | |
| | | (40) | | (35) | | (35) | | (35) |
| | P-14 | | P-05 | | P-05 | | P-05 | |
| | | (0) | | (30) | | (30) | | (30) |
| | P-12 | | P-06 | | P-06 | | P-06 | |
| | | (0) | | (40) | | (40) | | (40) |
| | P-13 | | P-13 | | P-13 | | P-14 | |
| | | (45) | | (0) | | (0) | | (0) |
| | P-04 | | P-14 | | P-12 | | P-12 | |
| | | (20) | | (0) | | (0) | | (0) |
| | P-03 | | P-12 | | P-14 | | P-13 | |
| | | (20) | | (55) | | (45) | | (45) |
| | P-02 | | P-04 | | P-03 | | P-03 | |
| | | (45) | | (20) | | (20) | | (20) |
| | P-06 | | P-03 | | P-02 | | P-02 | |
| | | (60) | | (20) | | (0) | | (0) |
| | P-01 | | P-01 | | P-01 | | P-01 | |
| Off-specs (MT) | 265 | | 200 | | 200 | | 200 | |
| Change profit (won) | – | | +21,600,000 | | +20,440,000 | | +22,117,000 | |

( ): change generation of off-specs

whereas B&B shows some troubles for certain kind of constraints.

## CONCLUSION

A decision problem of optimal production sequence in the petrochemical plant was investigated. A polyethylene product plant was selected as an example and Traveling Salesman Problem (TSP) was applied to solve the decision problem. Three well-known computation techniques such as Branch and Bound (B&B) method, Dynamic Programming (DP) method and Hopfield Neural Network (HNN) method were employed. The primary objective in the production sequence is to minimize the amount of off-specs so that the profit be maximized while satisfying constraints. The production sequences obtained by the present methods were compared with the sequences employed in the actual plant. From the comparison the sequences obtained in the present study showed reduced off-specs and increased profit. Effects of various constraints on the production sequence are to be investigated.


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## NOMENCLATURE

$C_{ji_{n+1}}$ : off-spec amount when operation j precedes the ending dummy operation
D : distance matrix
$D_r$ : reduced form of distance matrix
$d_{ij}$ : distance from city i to city j
E : energy function in neural network
$F_J(j\sigma_{J-1})$ : cumulative distance at the J th stage if operation j precedes the partial sequence $\sigma_{J-1}$
$\overline{F}(ji, i\,\sigma_{J-2})$ : $C_{ji} + \sum_{i,k \in \sigma_i} C_{ik}$
$I_{xi}$ : external input signal at tour x, order i
i : grade number
$i_0$ : index for the dummy (starting) operation in DP
$i_{n+1}$ : index for the dummy (ending) operation in DP
J : stage index in DP
LB(X) : a lower bound on the cost of tours of X i. e., $Z \geq LB(X)$ for x a tour of X
M : coefficient larger than the number of cities
N : total grade number
n : number of cities
P : total pure profit
PC : production cost
P − i(i=1, 2, 3, ⋯, 16) : grades of polyethylene products
PV : product value
PQ : production quantity
S : reducing constant
TCPQ : type change production quantity
u : input data in neural network
v : output data through neuron
$w_{ij}$ : weight connection between neurons
X, Y, $\overline{Y}$ : node
$x_{ij}$ : tour assignment
Z : total distance or cost
$Z_r$ : total distances to travel all tour under reduced matrix
{ } : sequence

### Greek Letters

$\alpha$ : positive constant or variable which controls the steepness of the sigmoid function
$\delta_{ij}$ : Dirac delta $= \begin{cases} 1: & i=j \\ 0: & i \neq j \end{cases}$
$\theta_{ij}$ : sum of smallest element row i, omitting $d_{ij}$ and smallest element column j, omitting $d_{ij}$
$\sigma_0$ : $\{i_{n+1}\}$, a set containing only the ending dummy operation in DP
$\sigma_J$ : an ordered set of J operation already sequenced in the last position in DP

### Subscripts

i, j : position or order of sequence in neural network x, y : tour in neural network method

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